MTH 1420, SPRING 2012
DR. GRAHAM-SQUIRE

LAB 7: SEQUENCES AND SERIES

Names: ____

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1. Instructions

(1) Introduce yourself to your lab partner(s), if you decide to work with someone.
(2) Work on the problems together with your partner for the remainder of the lab time. If you are confused about something, talk to your lab partner and explain your question to them to see if they can help. If everyone in the group is stumped, come talk to me for a hint. If you do not finish, it is okay to split up the remaining parts and work on them individually. However, you should meet up sometime outside of class to check each other's work before you turn in a final draft next week.
(3) Your group should write up and turn in one completed lab at the start of the next lab period. You can use this sheet as a cover sheet for the lab you turn in. **Each member of the group should write up at least part of the lab**, but you should check each other's work since everyone in the group gets the same score.

2. Introduction - New Sequences from Old

We have started to look at sequences, and if you have a sequence there are ways to build new sequences out of the original one. We will look at two ways of doing this: first looking at the difference between terms of the sequence, then looking at what happens if you add up the terms of a sequence.

3. Sequence of Differences

Given a sequence a_n, you can form a new sequence b_n by simply taking the difference between each two terms of the sequence. Thus $b_1 = a_2 - a_1$, $b_2 = a_3 - a_2$, $\ldots$, and in general $b_n = a_{n+1} - a_n$. We will call b_n the "sequence of differences" (Note that this is a term I just made up, it is not a general mathematics term).

Exercise 1. Calculate the first five terms of the sequence of differences b_n for the following sequences, and state if you think b_n converges or diverges (if you can prove it, do so).

(a) $a_n = n^2$
(b) $a_n = n$
(c) $a_n = \frac{1}{n}$
(d) $a_n = \frac{1}{2^n}$
(e) $a_n = \{2.9, 2.99, 2.999, 2.9999, \dots\}$
(f) $a_n = \{1, 0, -1, 0, 1, 0, -1, 0, \dots\}$

Exercise 2. Based on your results from the first exercise, can you make any conclusions about the relationship between the convergence of the original sequence and the sequence of differences? What are the relationships, and why do you think they are true?

4. Sequence of partial sums

Instead of subtracting subsequent terms to make a new sequence, one can look at the sequence s_n formed by *adding up* terms of a sequence a_n. Thus $s_1 = a_1$, $s_2 = a_1 + a_2$, $s_3 = a_1 + a_2 + a_3$, etc. Using sigma notation, we can write this as

$$s_n = \sum_{k=1}^{n} a_k$$

and we call this sequence the <u>sequence of partial sums</u> for the sequence a_n.

Exercise 3. Calculate the first five terms of the sequence of partial sums s_n for the following sequences, and state if you think s_n will converge or diverge.

(a) $a_n = n^2$
(b) $a_n = n$
(c) $a_n = \frac{1}{n}$
(d) $a_n = \frac{1}{2^n}$
(e) $a_n = \{2.9, 2.99, 2.999, 2.9999, \dots\}$
(f) $a_n = \{1, 0, -1, 0, 1, 0, -1, 0, \dots\}$

Exercise 4. Based on your results from exercise 3, can you make any conclusions about the relationship between the convergence of the original sequence, the sequence of differences, and the sequence of partial sums? What are the relationships, and why do you think they are true?

5. Series

The sequence of partial sums will be the basis for the material we cover in Chapter 8. In particular, we are interested in whether or not a sequence of partial sums diverges or converges, and what it converges to. If you take the limit as n goes to infinity of a sequence of partial sums, you get something called a *series*, which we usually denote with sigma notation as follows:

$$\lim_{n\to\infty} s_n = \sum_{k=1}^{\infty} a_k$$

Thus a series has two built-in sequences: The sequence $\{a_k\}$ and the sequence of partial sums $\{s_n\}$. What we will be studying in the near future is the relationship between properties of the original sequence and the convergence or divergence of the sequence of partial sums.

Exercise 5. Consider the series (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n}$ and (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n$. Calculate the first ten terms of the sequence of partial sums and make a guess as to whether the series converges or not. If it converges, what does it converge to?

Exercise 6. The following series all converge. Calculate the first ten terms of the sequence of partial sums and make a guess as to what it converges to.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{6^n}$

(d) Based on what you found above, what will the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{c^n}$ converge to if c is a positive integer greater than or equal to 2? What will happen if $c = 1$? What if $c = -4$?